Uncertainty Analysis of Reservoir Sedimentation

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Abstract

In estimating reservoir sedimentation, a number of uncertainties arise. These are related to annual streamflow, sediment load, sediment particle size, trap efficiency, and reservoir operation. Monte Carlo simulation is used herein to quantify the uncertainty of reservoir sedimentation. The proposed procedure has been applied to the Kenny Reservoir at White River, Colorado. The results show that streamflow and sediment inflow are the most important factors determining the uncertainty of reservoir sedimentation. The effect of each uncertain factor taken individually and taken in combinations, on the uncertainty of accumulated sediment, have been examined.

1. Introduction

Predicting the incoming sediment into a reservoir, its deposition, and accumulation have been important problems in hydraulic engineering. Despite the advances made in understanding the factors involved in reservoir sedimentation, its prediction is still a complex problem. Empirical models, based on surveys and field data have been developed to estimate reservoir sedimentation and accumulation (Strand and Pemberton, 1982). Likewise, mathematical models have been developed based on motion and continuity equations for water and sediment (Chen and Lopez, 1978). However, the empirical methods are still widely used in practice (Butler, 1987; Frenette, 1992). The main factors affecting reservoir sedimentation are: (1) quantity of streamflow, (2) sediment inflow, (3) sediment size, (4) specific weight of the deposits, (5) trap efficiency, and (6) reservoir size and operation. If some of these factors are uncertain, then reservoir sedimentation will be uncertain too.

Several methods of uncertainty analysis, such as First-Order Analysis (FOA), Monte Carlo Simulation (MCS), and Latin Hypercube sampling (LHS), have been

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developed and applied in water resources engineering (Yen and Tung, 1986; Salas, 1993). In this study, uncertainty analysis based on MCS is conducted to estimate the probability distribution of reservoir sedimentation (RS), and accumulated sediment volume (VRS). Annual flows are generated by a stochastic model. The effect of parameter uncertainty in the stochastic model on VRS, is also considered.

2. Estimation of Reservoir Sedimentation

In estimating annual sediment load, it has been common practice to use annual sediment rating curves for both suspended sediment and bed load. A method for determining annual sediment rating curves involves calculating daily sediment loads from daily sediment rating curves, adding them to obtain the annual load, and plotting the annual sediment load versus streamflow. Instantaneous sediment rating curves can be used as a substitute of daily sediment rating curves (Colby, 1956). Annual rating curves of suspended sediment and bed load may be expressed as

$$\log_{10} QS_t = a_1 + b_1 \log_{10} QW_t \tag{1}$$
$$\log_{10} QB_t = a_2 + b_2 \log_{10} QW_t \tag{2}$$

where QS_t = annual suspended sediment load (tons/day) in year t, QB_t = annual bed load (tons/day), QW_t = annual streamflow (cms), and a_1, b_1, a_2, and b_2 are rating curve coefficients for both annual suspended sediment and bed loads, respectively. Then, the total sediment inflow in year t is $QT_t = QS_t + QB_t$. The sediment which will be trapped in the reservoir can be estimated based on the trap efficiency of a reservoir. A formula relating the trap efficiency versus the ratio of reservoir capacity and annual flows is (Brune, 1953).

$$TE_t = a_3 + b_3 \{\log_{10} (C_{t-1} / IW_t) \}^2 \tag{3}$$

where TE_t = trap efficiency (%) in year t, C_{t-1} = useful reservoir capacity (m^3) at the beginning of year t, IW_t=31.536×10^6QW_t, streamflow (m^3), and a_3 and b_3 are regression coefficients. Then, the annual sediment RS(tons) trapped in a reservoir in year t is

$$RS_t = 3.65\, QT_t \times TE_t \tag{4}$$

and the accumulated sediment ARS_t in the reservoir (in tons) after t years is

$$ARS_t = ARS_{t-1} + RS_t \tag{5}$$

The sediment trapped in the reservoir will be compacted through time. The type of reservoir operation and the sediment size are significant factors affecting the specific weight of the deposited sediment. Miller (1953) developed a formula to estimate the specific weight W_t of sediments deposited after t years as a function of W_1, the initial specific weight, and K, a consolidation constant. Both W_1 and K are functions of the type of reservoir operation and the sediment size. Values of W_1 and K for various types of reservoir operation and sediment types are available. For a mixture of sediment, in which $P(c)$, $P(m)$, and $P(s)$ are the percentages of clay, silt, and sand, respectively, a weighted average of W_1 and K must be used (Lara and Pemberton, 1965). Thus, the volume of sediment deposited after t years of reservoir operation VRS_t (m^3) and the useful reservoir capacity C_t at the end of year t can be estimated by

$$VRS_t = 1000\, ARS_t / W_t \qquad (6)$$
$$C_t = C_o - VRS_t \qquad (7)$$

respectively, where C_o is the initial useful reservoir capacity.

3. Uncertainty of Reservoir Sedimentation

The uncertainty of reservoir sedimentation arises from a number of factors such as sediment inflow, water discharge, and sediment type. This section describes the methods used for analyzing and quantifying the various uncertainties involved.

3.1 Uncertainty of the Inputs

The various uncertain factors which affect reservoir sedimentation called here "stochastic inputs" may be categorized as: (1) those associated with annual sediment rating curves such as regression coefficients; (2) those associated with the type of sediment such as the percentages of clay, silt, and sand; (3) those associated with the equation for estimating the trap efficiency of the reservoir; and (4) those associated with water inflows to the reservoir.

The uncertainty associated with annual rating curves can be obtained from that associated with daily sediment rating curves. The parameters of the rating curves are uncertain because they are obtained from limited data. Well known results from regression analysis indicate that daily rating curve parameters a' and b' are bivariate normally distributed. Then, given that the uncertainty in the daily sediment rating curves can be determined, one can find the uncertainty in the annual rating curves by Monte Carlo simulation. The sediment entering into a reservoir is generally a mixture of clay, silt, and sand. Standard statistical analysis of field data may suggest a certain distribution function for predicting fractions of sediment type. The uncertainty of the trap efficiency regression equation (3) may be determined by estimating the means, variances, and covariance of the estimators $\hat{a}_3$ and $\hat{b}_3$.

Annual streamflow is an important factor affecting reservoir sedimentation. How to take into account the uncertainty of annual flows depends very much on the availability of data and the purpose of the simulation study. For instance, if the purpose of the study is to estimate the variability of annual sediment deposition in the reservoir on a yearly basis, one can express the flow variability simply by finding an appropriate probability distribution for the available flow data. Another alternative which is especially useful in determining the uncertainty of sediment accumulated in the reservoir through time, may be to analyze the annual streamflow data by stochastic methods. Stochastic time series models have been widely used in literature for many water resources problems (Salas, 1993). *AR* and *ARMA* models have been the most commonly used models for annual streamflow simulation (McLeod et al., 1978 ; Salas et al., 1980). The model parameters can be estimated from historical data. Often one uses the model to simulate synthetic flow traces assuming that the estimated model parameters are population values (known with certainty). However, since the model parameters are estimated from a limited sample size, the parameters are also uncertain and one may like to include such parameter uncertainty in simulating annual streamflow data. McLeod et al (1978) illustrated a procedure for incorporating parameter uncertainty in simulations based on *ARMA* models.

3.2 Uncertainty of Annual and Accumulated Reservoir Sedimentation

The uncertainty of annual reservoir sedimentation RS of Eq.(4) is discussed, herein. Specifically, one would like to determine the probability distribution of RS or the distribution of the corresponding volume. This can be accomplished as: (1) obtain a random set of size n of the stochastic inputs $\theta_i = \{ a_1, b_1, a_2, b_2, a_3, b_3, P(c), P(m), P(s), QW \}_i$, $i = 1,\ldots,n$ from their corresponding probability distributions by using Monte Carlo simulation, (2) use the procedure outlined in Section 2 to determine the corresponding annual sediment deposited in a reservoir RS_i, $i=1,\ldots,n$, and (3) analyze statistically the output RS to determine its probability distribution and its basic statistics such as the mean, variance, coefficient of variation, and skewness coefficient.

How much sediment will be accumulated in a reservoir throughout its expected life is an important aspect in the design and management of reservoirs. This is even more so when reservoir sedimentation is subject to a number of uncertain factors as discussed in the previous sections. The uncertainty of accumulated sediment in the reservoir can be accomplished by Monte Carlo simulation studies. First, stochastic inputs are generated by the procedures outlined in Section 3.1. Then, the set of stochastic outputs are generated, namely, the reservoir sedimentation RS_t in a given year t, and the accumulated sediment ARS_t which is then converted into volume VRS_t . Lastly, the generated output data is subject to statistical analysis.

4. Application

The procedure of uncertainty analysis of reservoir sedimentation described above was applied to the Kenny Reservoir, White River in northwestern Colorado. Taylor Draw Dam was constructed and filled in 1984, the reservoir capacity is 17×10^6 m^3 and the drainage area is 7,183 km^2 . Basic data was obtained from Tobin et al (1990). Annual streamflow data measured during 1983-1993 at White River below Boise near Rangely was extended to 30 years by simple linear regression based on data at White River below Meeker. The extended data was modeled by an AR(1) model with parameters: $\hat{\mu} \sim Normal$ (20.33, 4.90), $\hat{\phi}_1 \sim Normal$ (0.544, 0.0234), and $\hat{\sigma}_\epsilon^2 \sim Normal$ (30.54, 2.04). The daily rating curves were constructed by using 20 data for suspended sediment, and 18 for bed load. Using the daily rating curves and the historical daily flows during 1983-1993, annual rating curves for suspended sediment and bed load were constructed. A trap efficiency curve was made by using Brune's data. The information about the fraction of each type of sediment was extracted from Tobin et al.(1990). The statistical properties and their distributions of all stochastic inputs are summarized in Table 1.

Annual suspended sediment load, bed load, and RS load and volume, were simulated and estimated by using the method described in Section 3.2. Their statistical properties are shown in Table 2. Goodness-of-fit tests indicated the distribution of annual RS volume as Gamma 2. The uncertainty of accumulated reservoir sediment VRS throughout 60 years were estimated to see the cumulative effect and sensitivity of each uncertain factor as: (A) uncertainty from annual streamflow not including the uncertainty of model parameters, (B) uncertainty from annual streamflow including the uncertainty of model parameters, (C) uncertainty from annual streamflow and total sediment inflow, (D) uncertainty from annual streamflow, total sediment inflow, and trap efficiency, and (E) uncertainty from annual streamflow, total sediment inflow, trap efficiency, and the fraction of each sediment types.

Table 1.Statistical Properties of Stochastic inputs

Input	Lower bound	Upper bound	Mean	Standard deviation	Distribution
a_1			0.481	0.455	Normal
b_1			1.956	0.073	Normal
a_2			-0.068	0.438	Normal
b_2			0.784	0.248	Normal
a_3			99.5	1.541	Normal
b_3			-13.6	0.517	Normal
P_c	16	41			Uniform
P_m	39	63			Uniform
P_s	14	43			Uniform
QW			20.33	6.57	Normal

Table 2. Uncertainty Analysis of Annual Reservoir Sedimentation

	SS load (tons)	BS load (tons)	RS load (tons)	RS volume (m^3)
Mean	540,542	3,602	359,093	307,250
Standard Deviation	334,047	1,497	197,533	197,533
Coefficient of variation	0.618	0.414	0.619	0.643
Skewness coefficient	1.133	0.774	1.182	1.329
Distribution				Gamma2

The mean values of accumulated reservoir sediment volumes throughout 60 years for each case are shown in Fig.1(a). The results show that the uncertain factors increase the mean of VRS_t. Specially, sediment inflow is the most significant factor which increases the mean of VRS_t. Fig.1(b) shows the corresponding standard deviations of VRS_t. For example, the standard deviation of VRS_{30} (t= 30 years) was about 30% of its mean value. It indicates not only the significance of the amount of standard deviation, but also the importance of uncertainty analysis, in estimating reservoir sediment deposit volume. The results can be useful for determining the risk that accumulated sediment in the reservoir after some years can exceed a specified value.

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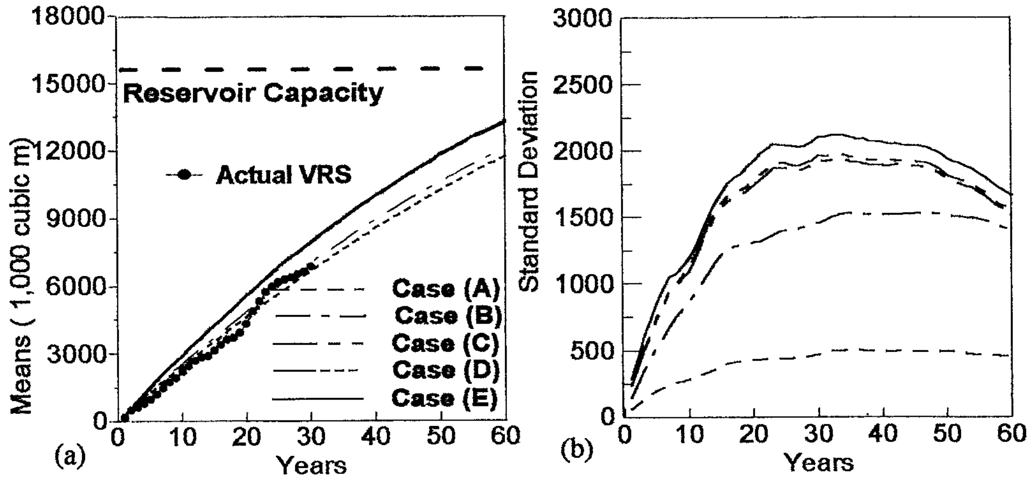

Fig. 1 (a) Mean and (b) Standard deviation of accumulated reservoir sediment volumes for five cases

Appendix I: References

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